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# ARITHMETIC FOR THE PHILIPPINE ISLANDS



BOOK I.

ANNETTE L. CROCKER



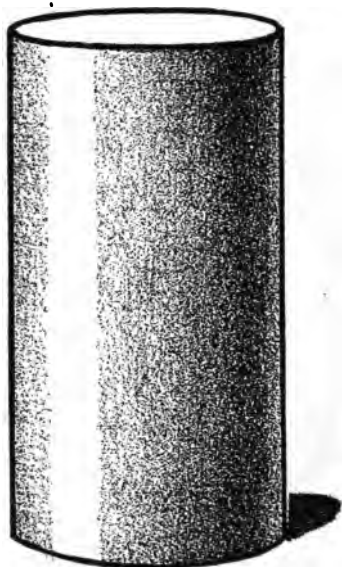
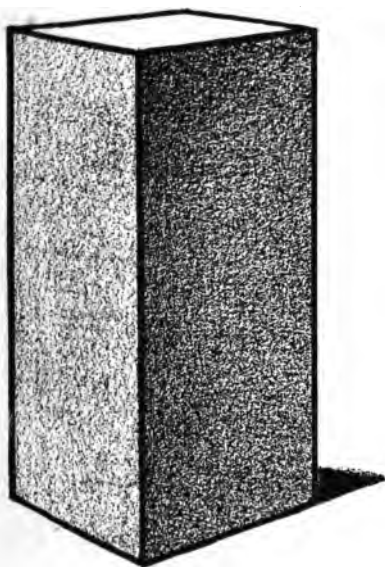
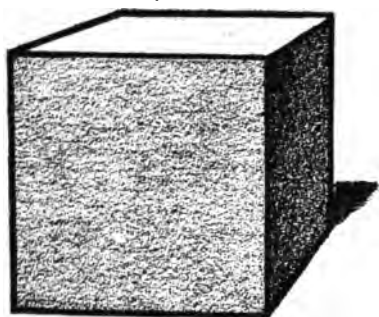
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**ARITHMETIC**  
**FOR**  
**THE PHILIPPINE ISLANDS**  
  
***BOOK I***



Katina  
9/15/08  
S. C.

# ARITHMETIC

FOR  
THE PHILIPPINE ISLANDS

BY  
ANNETTE L. CROCKER, A.B.  
FORMERLY TEACHER OF MATHEMATICS IN THE PHILIPPINE  
NORMAL SCHOOL

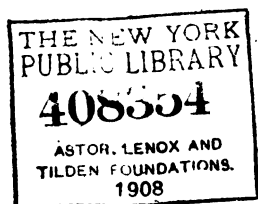
*BOOK I*  
TEACHER'S BOOK



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1906  
M.C.C.





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TO

THOSE FILIPINO TEACHERS

WITH WHOM IT HAS BEEN MY PLEASURE TO TEACH  
FOR FOUR YEARS, AND TO WHOSE FAITHFULNESS  
AND COURTESY MUCH OF MY JOY IN THIS TEACHING  
HAS BEEN DUE, THIS BOOK IS CORDIALLY DEDICATED



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# ARITHMETIC FOR THE PHILIPPINE ISLANDS

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## PURPOSE OF THE BOOK

THIS book deals with the arithmetic work of the First Grade.

It includes:

1. A brief outline of the work.
2. Talks with the teachers upon the purpose of this work and the ways and means of accomplishing it.

The purpose of this first year's work is threefold:

1. To do its part in training the children in English speech.
2. To develop the children's sense of magnitude.
3. To make them well acquainted with all the combinations of number through 10.

## OUTLINE OF THE WORK

The work is divided into nine steps, as follows:

### STEP I

1. Count in English from 1 to 5. .
2. Recognize the number of objects in groups.

## 3. Ask and answer simple questions in English:

How many ——— do you see?

"I see ——— sticks."

How many ——— has José?

"José has ——— balls."

How many ——— have you?

"I have ———."

## 4. Seat work:

Make outline pictures of 1, 2, 3, 4, 5, different objects.

Stick-laying for the youngest pupils.

## 5. Words to be learned and used daily henceforward:

one      three      five      have      see      I

two      four      has      you      how many

**STEP II**

## 1. Make figures to 10.

## 2. Recognize objects in groups to 10.

## 3. Ask and answer questions:

How many ——— are there on the window?

"There are ——— on the window."

"Carmen can make ———."

"Maria makes ———."

## 4. Seat work:

Draw outline pictures from objects and write figures beside them.

Give figures and let children choose and make pictures of their own.

Make figures.

5. New words:

six	eight	ten	are there	makes
seven	nine	can	there are	make

STEP III

1. Magnitude:

Distinguish between large and small objects.

See, name, feel and weigh different solids.

Find other solids of the same shapes at home or at school.

2. Express in English sentences differences in size and weight:

"Maria's ball is small."

"My ball is larger."

"José's ball is larger than that ball."

"This banana is larger than that banana."

3. Seat work:

Drawing with colors	} objects of different sizes.
Cutting	
Molding	

4. New words and phrases:

cube	large	light	larger than	Carmen's
sphere	small	larger	this	my
prism	heavy	smaller	that	is



## STEP IV

## 1. Equality:

- (a) Find equal objects.
- (b) Draw figures equal to other figures.
- (c) Build solids equal to other solids.

## 2. From the parts used in this building teach

$$\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{2}{2}, \frac{3}{2};$$

Teach 2 times, 3 times.

## 3. Count from 10 to 30 or to 50.

## 4. Questions and answers:

How many ——— make ———?

How many ——— equal ———?

What part of ——— is ———?

"José's cube is 3 times my cube."

"Carmen can build a large prism."

Are these equal?

"This is one half as large as that."

## 5. Seat work:

With bamboo sticks, seeds or shells, make lines and figures to illustrate these relations.

## 6. Words and phrases:

alike	build	times	times as large
equal	half	third	half as large
equals	halves	thirds	these

### STEP V

1. Review with solids, plane figures and lines, all the combinations previously studied.

2. Recognize at sight different magnitudes.

3. Teach  $\frac{1}{4}$ ,  $\frac{2}{4}$ ,  $\frac{3}{4}$ .

4. Seat work:

Use circles, squares, other rectangles and triangles cut from colored cardboard.

Have children combine these and make designs with them.

Review previous Seat work.

5. New words and phrases:

circle

square

oblong

triangle

fourth

fourths

### STEP VI

1. Addition and subtraction to 7 (37 combinations).

Develop with solids and plane figures.

2. Practice in hearing and answering problems.

3. Practice in reading and writing the arithmetic symbols

$$+, -, \times, =, \frac{6}{2}.$$

4. Seat work:

Copy and illustrate simple number combinations.

## 5. Words and phrases:

sum	take away	lose	buy	loses
remainder	less	pick	buys	picks
left	give	sell	gives	sells

**STEP VII**

1. Children make original problems.
2. Review of combinations through 6.
3. Seat work:  
Simple examples.

**STEP VIII**

1. Daily drill on combinations of numbers.
2. Develop combinations from 6 to 10 (62 in all).
3. Problems:  
Children state problems of their own.  
Teacher write problems on board for children to read.
4. Seat work:  
Examples in addition, subtraction, multiplication and division.

**STEP IX**

1. Teach dozen.
2. Teach linear measure, inch, foot, yard.
3. Daily drill on number combinations.
4. Daily applications in problems.

## 5. Seat work:

Examples as in Step VIII.

Make figures of certain definite sizes.

Copy, fill out and answer incomplete problems.

## 6. New words and phrases:

inch	foot	dozen	worth
inches	feet	yard	twice

**TIME NECESSARY FOR THE WORK**

Children who enter school at five or six years old will probably spend more than one year on these nine steps. On the other hand, the older children who do not begin studying in English until they are eight years old, or older, may well cover this ground in much less than one school year. The age and capacity of each class must determine the length of time it needs to complete the work.

**MATERIALS NEEDED**

The materials that are needed are indicated from time to time. With one exception, they are such as may be obtained in any town in the Philippines.

The children will have no books; they will depend upon the teacher to guide them each day in their work.

Sticks of split bamboo are very useful for counting, for stick-laying, and, when they are cut in different lengths, for teaching  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{2}{3}$  and 2 times, 3 times, etc. Fruits, seeds, shells, and bamboo balls will all help in this work.

Colored paper can be found in any Chinese shop. Out of

this can be cut squares, circles, and triangles; and these can be used for teaching number relations and also in the Seat work for making designs.

In addition to these home materials I hope that each school may have a set of wooden blocks made in the following shapes and sizes. These blocks can be made in the provincial trade schools and will be useful in all the grades of the primary work. Each set should contain:

20	4 cm. cubes.								
1	8 cm. cube.								
1	12 " "								
6	prisms, 4 cm. sq. on the base and 8 cm. high.								
4	" 4 " " " " " 12 " "								
1	prism 4 " " " " " 16 " "								
1	" 8 " " " " " 4 " "								
1	" 12 " " " " " 4 " "								
4	cylinders of 5 cm. diameter " 4 " "								
2	" " " " " " 8 " "								
1	cylinder " " " " " 12 " "								
1	sphere 4 cm. in diameter.								
1	" 8 " " " "								
4	hemispheres each 4 cm. in diameter.								
4	" " 8 " " "								
4	quarter spheres each 4 cm. in diameter.								
4	" " 8 " " "								
<hr/>									
60									

The use of these 60 blocks in the class room will help more than anything else to cultivate in the children a definite sense of relative magnitudes.

## INTRODUCTORY TALK TO THE FILIPINO TEACHERS

---

WHEN your pupils first enter school what do you wish to teach them?

“Reading,” perhaps some one will say, or “English” or “Arithmetic.”

I think all of you are right, and that what we need at first is to teach our young pupils

how to read,  
how to write,  
how to use numbers,  
how to speak English.

In this book I wish to plan with you a way by which we may teach our pupils how to use numbers. We may carry out the plan in oral lessons with all our beginners, and I think it will fit them to study intelligently the text-books in arithmetic that we shall give them later. This book is not, of course, then, for the children; it is for us teachers alone.

When your children first enter school, they are five, or six, or seven years old, are they not? And they know very few words in English, and they have not thought at all about questions in arithmetic? We must remember all the time that these children are learning a new language, and we must encourage them in every way to use this language in the ex-

pression of their thoughts. It is only as we make the use of the English phrases and sentences a daily custom in all their classes that we are able to make our pupils speak English naturally and freely.

This need of using continually the English language is an important one. More important still, however, is the necessity of leading our children to think for themselves. It is one thing for you to have ideas of your own about arithmetic and to say them over and over to your class until they are able to repeat them after you. But it is quite another thing for you to lead the child to see something for himself and, after he has seen it and has felt what it means, to encourage him to tell you about it. It is only as you lead the child to express what he himself can discover about the relations of things about him, that you really teach him. In the other way you only hear memorized recitations, and the hearer of such recitations is not truly a teacher.

We have a good rhyme in English that says:

“Count that day lost whose low descending sun  
Views from thy hand no worthy action done.”

For us teachers this might read:

“Count that day lost whose low descending sun  
Views in thy class no new idea won.”

What are all of these little children doing outside of school hours? They are playing, are they not? swimming, running, jumping, playing games—continually in action? Now we cannot expect them to enter the schoolroom and sit still for hours on long benches. Activity is the first law of all life.

It is only dead, lifeless things that are inactive; and what we want to do in our schools is not to deaden our children's faculties, but to quicken them and so make the children more alive to the facts and relations of the world they live in.

This activity that the child brings to school as a part of himself is itself a product of his development. The infant child has very few activities. It is only as he lives on and grows that he becomes able to walk and talk and run and reason. What we teachers have to do when the child enters school is to direct his activity and regulate it so that with its aid he may learn something of the reading and writing and arithmetic that we want to teach him.

Perhaps some of you think I mean to have the children noisy in school. The best regulated school is the one where the children are always busy with hands and mind. It is only the idle child who has time for noise and mischief. And the best teacher is the one who always has something for everybody to do.

You will probably have some older children entering your school with the younger ones. Whenever it is possible I like to make a separate class of these older ones. Just because they have lived longer they are generally more developed in other ways and can progress more rapidly in their studies than the little children can. But it is best, I think, whatever their ages, to begin the arithmetic in the same way with all beginning classes.



## STEP I

LET us begin with counting. You cannot find a child in the class, can you, who will not know how to count in his native language? We will teach him then how to count in the new language. Count at first only from one to five. Bring some objects to school; some things that the children all know, or, better still, some things that the children can get for themselves. In every place you can get pieces of young bamboo. Cut them into sticks as long as this:

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One decimeter.

This makes a good length for the children to handle; and if you cut the sticks in a metric length, they will be useful later on in the work.

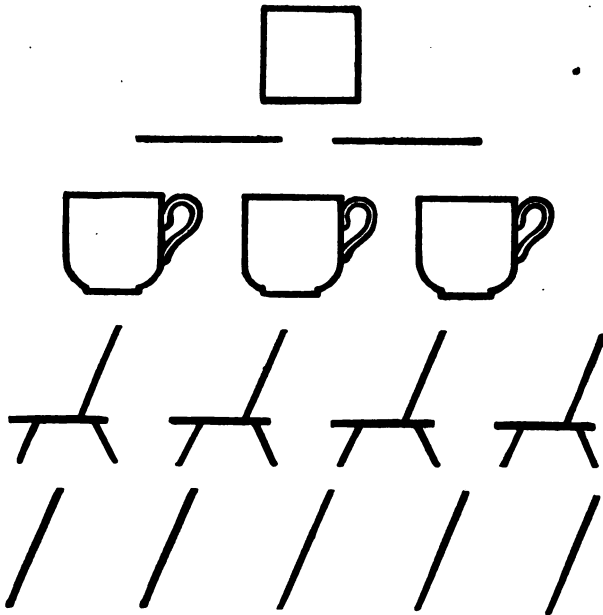
Bring flowers. Children like pretty things as well as you do. And you may find shells and seeds and fruits, or you may bring leaves, or use books and pencils and balls—anything that the child may handle and count by actual contact.

Whatever you choose, hold up the objects and let a child count them as you place them one by one on your desk. Say the new English words with him the first time, and then let him try to count the objects alone. Let another child count them in the same way; then another. Are all the children

in the class watching and listening? Be sure that each child who counts speaks loud enough for all the others to hear him.

You do not think, do you, that I mean to have the children shouting? If the class is interested and attentive and the child who is reciting speaks in the same voice he uses when at home, he can be heard by every one in the class.

After five or six children have counted the objects as you put them on your desk, let one child come forward and hold up the objects while another child counts them. Vary the work also by making lines or outline pictures on the board, like this:



Question the children as you are making the pictures. Have them tell you what you are making. This stimulates their interest; and because the teacher makes pictures, they will be eager to try to make them too. Let the children count the pictures and tell you how many squares or cups or chairs they see.

Be careful here about the pronunciation of the new words. These children are very quick to hear; and if only we teach them carefully, they can learn to speak English words as well as any American child can.

Teach every child to sound the *v* in *five*. If we allow a child of six to say "fi," he will keep on saying it and some other teacher afterward will have to work doubly hard to break him of this bad habit. I have had many pupils—pupils who have studied English two or three years—who have said "fi" and "twel" and "sick," and I have had to work very hard with them to make them overcome these faults. Yet I know by experience that the youngest pupils can be taught in a few days to say these words correctly.

Let your pupils learn the new pronunciation naturally. Do not make them repeat the words over and over; but whenever a child mispronounces a word, let him hear that word spoken once correctly. The little child does not know why he should say "five" rather than "fi"; but all children are imitative animals, and if you keep on saying "five" they will in a short time hear and adopt the correct sound.

Be sure that you always say "five" and "six" and "twelve," yourself. If some one child continues to make the same mistake in pronunciation, let some other child who speaks the

word correctly say it for him. Have you ever noticed how much more quickly children will learn sounds from other children than from you? Remember this, and it will help you all along. It will help in two ways: The child who is doing for the others is in action and is making himself more sure of what he knows, and the children who are listening are being stirred to similar action by the desire to do as well as the other boy can.

These three numbers, *five* and *six* and *twelve*, are among the words most commonly mispronounced by the Filipino children. Let us start our pupils right with these. Besides these, there is the question of plurals. Every child who comes to school is able to see the difference between one object and two or three or more objects and so to understand the meaning of the plural forms, and certainly every one of them is able to sound the letter *s*. It is only inattention on our part then, that allows boys and girls to enter the high schools saying "book" and "pencil" when they mean "books" and "pencils." Insist upon correct speech here in the same way that you do for the troublesome numbers, and do not be satisfied with your work until your pupils are speaking correctly.

This work has been training the children to pronounce the English names for the numbers and to count in order. Now let us begin to help them to see and call by their number a group of objects. Hold up 3 sticks and let some child tell how many. Have him answer quickly. The purpose is, not to count them, 1, 2, 3 sticks, but to make the child see at once the three objects and recognize their number.

Hold up groups of 2, 3, 4 or 5 other objects and have the children name their number, in the same way.

Have some child come forward to your desk and let him hold up a number of sticks, while some one of the others answers the question, How many sticks has José? This will help the others to express in English what they see; and after two or three days' work, the children will be able to tell you in good sentences,

“José has three sticks.”

“Carmen has four pencils.”

This expression in English of what the children are doing or seeing is most important. While we are teaching the children arithmetic, we are also teaching them English; and the English expression of what they know is something that is quite as important as the arithmetic. Make them see and hear and understand. Then make them express in English words what they know. Help the children along by asking them questions, but have them express their own answers in the words they can use. Do not say the answers to them and have them repeat them after you, word by word. Parrots can do this, but we want our pupils to do more than this.

This is ground enough for one lesson. Perhaps it is too much for the little ones; for you should not keep the little ones busy at one subject for more than twenty minutes.

In this lesson we have tried to have the children do three things:

1. To count in English from one to five.
2. To see and call by number groups of objects.
3. To tell in a sentence the number of objects that they see, or that some one has.

Your class cannot learn to do this in one day. It may be two or three weeks, or, with children of five and six, a longer time before your class can answer promptly your questions.

Give them work like this each day, but make it varied and more interesting to them by using different objects and by changing the outline pictures.

In some of the later lessons, let the children take turns in asking questions of each other. They will like to do this, and it will help them to learn the form of the question in English. In this question and answer work, too, they can learn by practice how to use the words *have* and *has*.

*One, two, three, four, five*, also *see, have, and has*, are the words we want the children to learn thoroughly in these lessons, by using them naturally. The names of the objects they use in the class room and the pronoun *I*, they will acquire unconsciously. These other words we want them to understand and then to use daily until the words belong to them, until they use them as freely as they use the words of their own dialect.

We will call these first lessons in arithmetic, that I have just been describing, the *First Step*. The word *lesson* always makes us think of something we can do in one day; and the word *step*, I hope, will make you think of something that has to be done before one can advance farther.

A fifteen or twenty minute lesson in arithmetic is quite long enough for children just entering school. But with this lesson we may give them each day another twenty minutes for studying about numbers. This studying we can call their *Seat work*. Do not give them their number lesson and their

Seat work one after the other. Put the lesson before recess and the Seat work after recess; then each part will seem new and fresh to the children.

For the Seat work for the First Step let each child make on paper or on his slate or on the blackboard outline pictures of objects. Draw the pictures on the board and have the children copy them. Give different pictures each day. Make the pictures very simple. If the children are very young, give them sticks of bamboo of different lengths and let them lay the sticks on their desks in these shapes. Have the children make these pictures as neatly as possible.

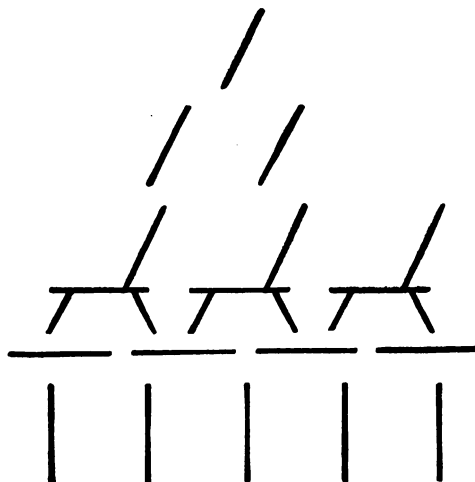
On pages 19, 20, and 21 are outline pictures arranged for the Seat work for one week. These pictures are all made of straight lines so that they may be used equally well for models for stick-laying. For this you will need many sticks. The pictures I have arranged here need sixty sticks for each child. Ask the children to bring sticks. They will be glad to help and, as I have found, will come with their handkerchiefs full of lengths of split bamboo or of some soft wood. Cut the sticks of two or three different lengths so that they can be used in the different parts of the figures. Make them, for instance, 5 cm. and 10 cm. and 15 cm. long. Can you not color some of the sticks red or blue or pink or yellow or green? If you can, the colored sticks will be sure to delight the children, and you can give them to those children who do the best work.

Put the figures on the blackboard and have the children copy them with their sticks. After they have laid the figures neatly on their desks, you can well use them for a conversation lesson. Each child will be glad to tell you how many chairs

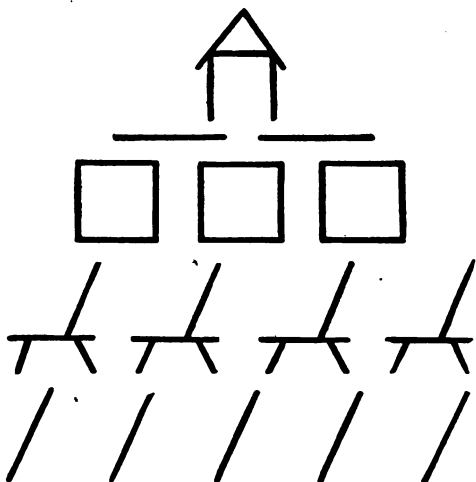
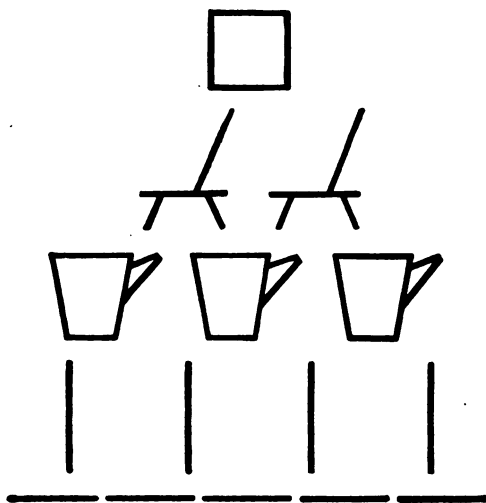
or hats he has made or to look at his neighbor's work and tell you how many José or Maria has. They can count the sticks in each figure too, and tell you how many sticks they have in a hat or a house.

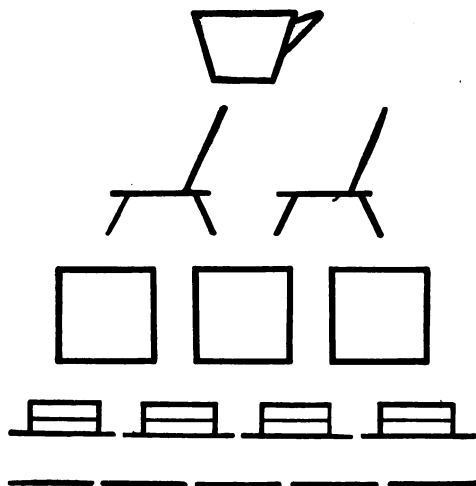
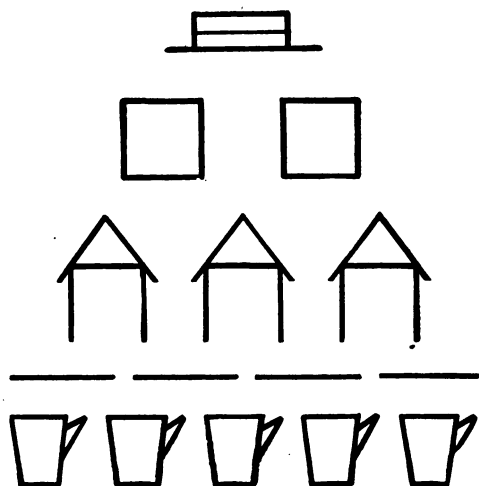
After they have been laying sticks for some days, vary the Seat work by putting outline pictures on the board and letting the children copy them on their papers or slates. You will find other suggestions for this work in the two pages of outline drawings at the end of this book.

When one child does very well, pin his paper on the wall so that all the children may see how well he has done. Then they too will try to do as well.



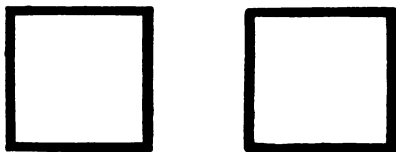






## STEP II

THE children have been saying the numbers and learning to recognize the number of objects in different groups, but they have not yet learned how to make the figures that we use to represent these numbers. These we may now teach them. Make outline pictures yourself on the board and let the children tell how many pictures you have made. Then make the figure after the picture.



How many blocks have I made?

"Two blocks."

This means two. 2.

Can you make two, José?

What does José make?

"José makes two."

Can you make two?

"I can make two."

*Can* is a good word to teach them; for it is what *can* expresses—the ability to do or to think—that we want to teach

our children. The boy or girl who *can* is the boy or girl who will have power.

Talk with the children all the time about what you are doing with them, but give them opportunity always to frame their own answers.

Make the figures separately and let the children tell the name of each. Let them find 7 girls or 2 books or 3 flowers and write on the board the number that tells how many.

Bring new objects often to school. The children will like to talk about the familiar fruits and flowers and will be eager to learn the English names of the objects around them; and the variety will hold the children's interest from day to day.

In this step teach the children to count through 10 and to recognize any number of objects, to 10, that you may place in a group.

Give small groups of objects to the different children and ask first one and then another to show you different numbers of objects. Let others tell how many these have, as

"Carmen shows 8 beans."

"José has 6 sticks in his hand."

"There are 5 flowers on the window."

"Maria puts 3 bananas on the table."

Vary your questions by calling attention to the position of the objects.

How many chicos are there on the window?

"There are 3 books on the desk."

Question them, too, about the number of different objects they see outside of school—in their homes, or in the market, or

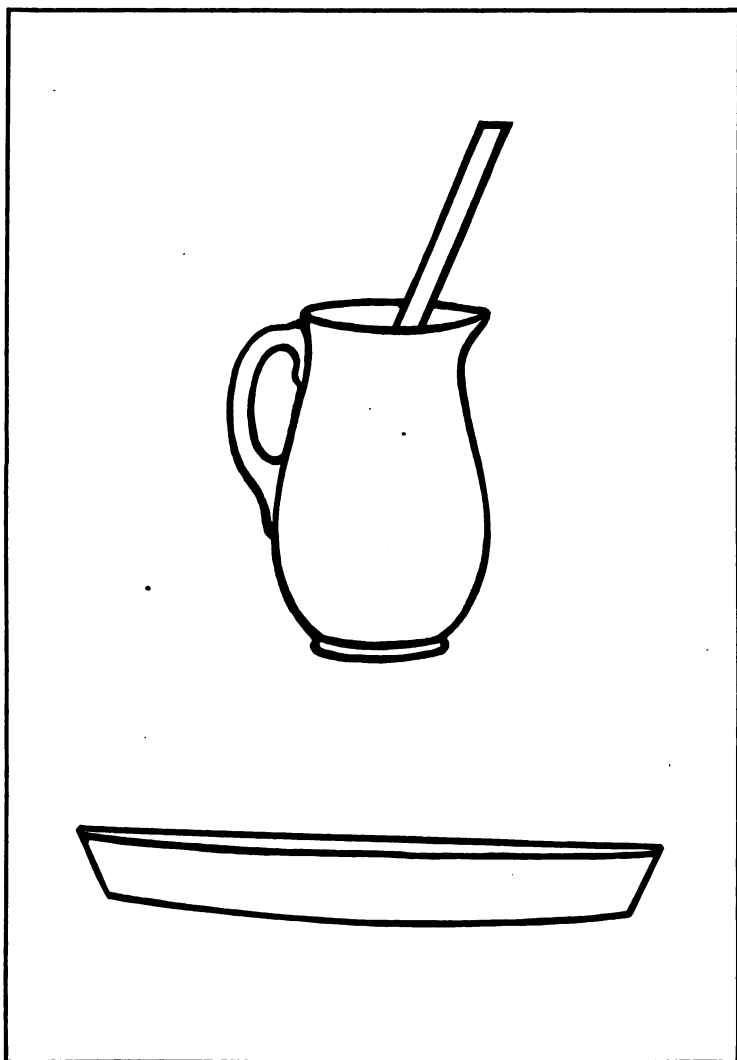
on the way to school. This questioning ought to encourage their habits of observation and help to keep them speaking English.

Does this seem very slow to you? But remember that these children are learning about numbers and learning a new language at the same time. And if they learn thoroughly these first steps, their progress afterward will be rapid.

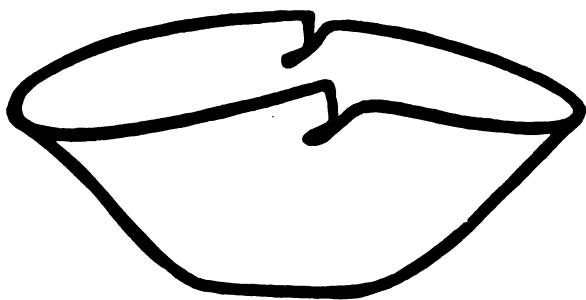
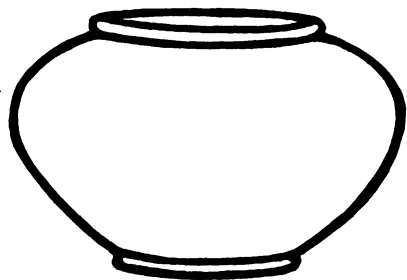
For Seat work in this step the children can make pictures of objects and place beside them the figure that tells their number. Give them here the objects themselves. The outline drawings on the following pages are illustrations of familiar objects, and the children can make pictures of all of these. It is much more fun to the child to make an outline picture of the ball or stove or banana itself than to copy your picture of it. And his drawing, however crude it may be, will be the child's own expression of what he sees, not a mere copy of what you have seen for him.

Choose familiar things—fruits, or dishes, or objects that they know well. Choose ones, too, that have simple outlines. Put these things on your desk or in some other place where all the class may see them, and ask the children to make pictures of them. (If you have colored pencils, let the children take turns in making colored pictures.) Give them certain numbers on the board and let them make their own pictures to fit them. Give the numbers to them in different order each day. One day give them 4, 3, 1; another day give them 2, 5, 3, 4. Occasionally also let them make the numbers themselves, very neatly and in order:

1   2   3   4   5   6   7   8   9   0

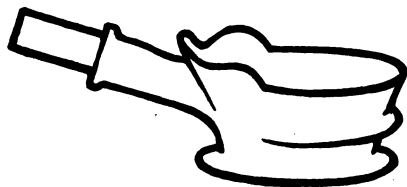
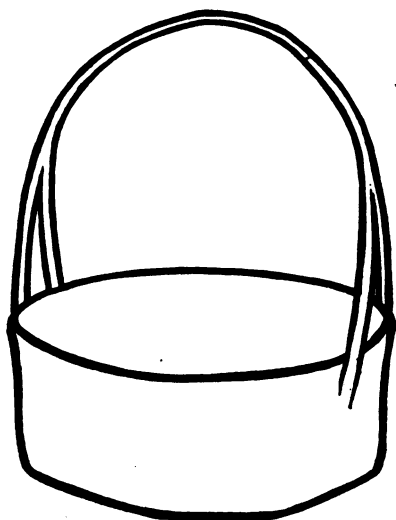


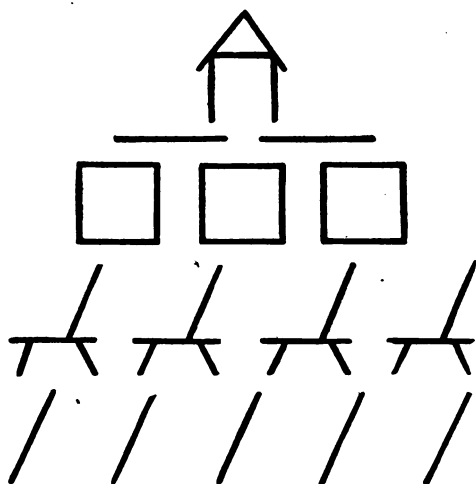
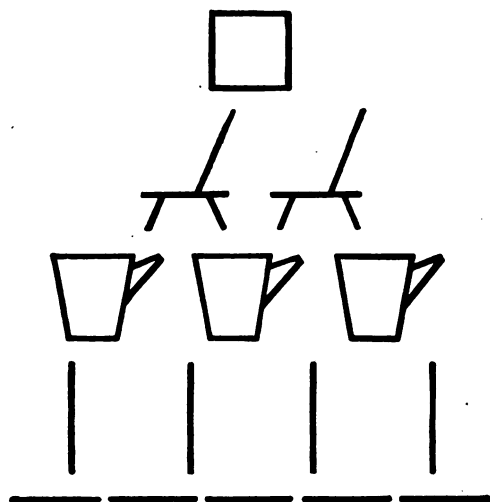


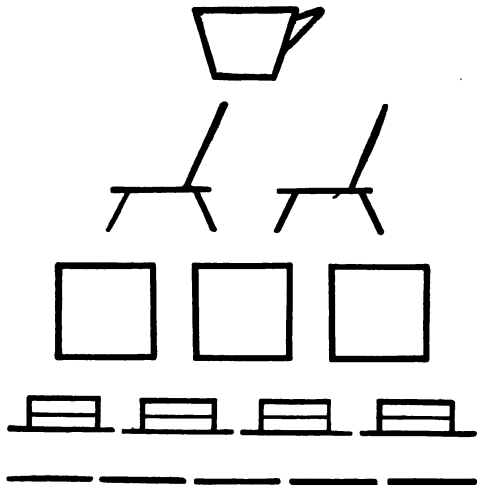
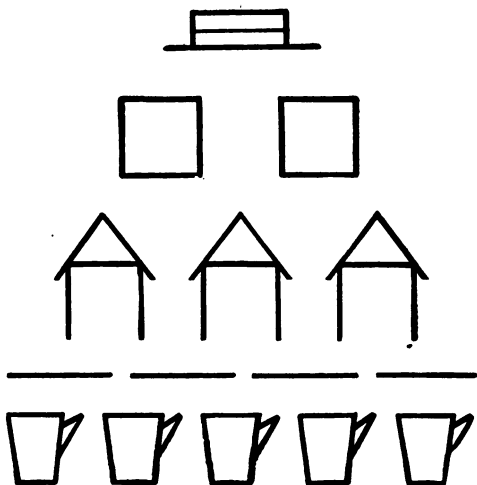












for that we must make our pupils comprehend. The measuring facts—the sense of the size and the shape and the weight of objects—in a word, the sense of the magnitude of things, is the basis of arithmetic.

The Filipino children do not understand about the magnitude of things. Some of you older people, too, lack training here. I have had grown people tell me when I asked them the distance from their native town to the next pueblo, that it was “an hour’s ride in a carromata.” This time measure is a kind of measure, it is true, but at best it is a very crude kind. The distance between two towns should be measured in distance units. “An hour in a carromata” is a rough measure of speed; but speed and distance, though one helps to pass over the other, are not the same thing.

We need to make the child realize the values of the numbers he can now say so well, and for two reasons:

1. Because this measuring idea is at the foundation of arithmetic.
2. Because our pupils at present have their measuring sense undeveloped.

I want you to put especial emphasis upon this part of the work. We want to make the child picture clearly in his mind the value of each number that he uses. He must realize that 2 represents something two times as large or as heavy as the thing represented by 1, or maybe something only one-half as large or as heavy as the thing represented by 4. It is the value of one thing when compared with another that we wish to make him comprehend.

This object is 3 times as heavy as that.

This pencil is only half as long as Maria's pencil.

Do not be afraid of the fractions; if the children really understand the difference in magnitude, they can express it as well by saying a fraction as by saying a whole number.

The children can now count objects together and can easily learn to tell you that 2 and 3 are 5, etc. They could also quickly memorize the addition and multiplication tables, but this would not help them at all to understand the magnitudes that figures represent.

We want then to lead the children to understand the value of the numbers they are saying. How can we accomplish this? It is something practically new to the child. How do we ourselves learn about any new thing, something that we have never seen or thought of before? We must see it, mustn't we? or touch it, or taste it, or smell it, or hear it? With the children, too, the five senses are the quick messengers that carry new impressions to their minds. We cannot make them smell or hear arithmetic, but their touch and sight will aid us; and if we develop them as we should, they will repay us well. Only we must not expect too much at first from them. The children are young and untrained and their senses and their power of observation are untrained, too.

Our aim in this Third Step is to make the child observe and realize difference in size and shape and weight. Later on we will let him find out the exact differences and measure them; but do not here try to make him see more than that one object is large and another is small, that this one is larger than that.

Give the first lesson from objects that you can find in your own neighborhood. Bring bananas and chicos and balls of different sizes. Let the children take them in their hands and notice who has the large ones, who has the small ones. Encourage them to talk about these objects. .

“Yes, Carmen has the large ball. I have the small ball.”

When they have learned to notice and express this difference, let them compare two or three objects and find by observation the use of our words *larger* and *smaller*. They know these objects, and so they can fix their attention upon their sizes and upon expressing what they learn in English.

These objects that you have been using cannot be measured exactly. No banana or camote is exactly twice as large or one third as large as another. We shall soon need to use objects that can be measured exactly; that is, objects that have been made by definite measurement. Solid objects—spheres, prisms, cylinders—are the best for this purpose. I wish that each school may have a collection of wooden solids like the ones pictured in the frontispiece and described on page 8. If you have them, begin to use them now. These are new shapes, so let the children first learn to know them.<sup>1</sup>

Give the class first a sphere, a cylinder, a cube, and a prism. Have the children take them in their hands and feel the difference between the shapes of the cylinders, cubes, and prisms. Let them see by actual trial that the sphere can roll

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<sup>1</sup> If you have no solid figures and cannot obtain them, you may begin here to use the cardboard figures, the squares, circles, and oblongs that are mentioned in Step V.

every way, that the cylinder can roll only one way, and that the cubes and the prisms cannot roll at all.

Have them find all the other things they know that are the same shape. Their quick eyes will discover that the ball and the orange are like the sphere, that the chalk and the bamboo sticks and, perhaps they will say, the banana too are like the cylinder. And surely they will discover that the chalk box and the wooden posts are prisms. Have the children notice the objects in their own homes, and tell you about the ones that have these shapes.

It will take several days perhaps to do this. After the children have learned the different shapes and can talk about them naturally, telling you, "My pencil is a cylinder," "My orange is like a ball," "The —— is a prism," etc., lead them to compare the sizes of the different shapes.

Distribute the blocks among the children, and they will easily tell you about each one.

"My cylinder is small."

"José's cylinder is larger than my cylinder," etc.

They can easily discover, too, which one is heavy and which one is light; and when they have discovered that fact for themselves, they will be eager to tell you about it.

The aim in this step is to lead the children to observe different magnitudes and then to express in English what they have learned. We have not attempted any exact comparison. Children do not at first see how much larger or smaller one object is than another, but they notice the general fact that this is larger or smaller than that.



For Seat work let the children make outline pictures of larger and smaller objects. Let them draw from objects and make larger and smaller pictures. If you have colored pencils or crayons, let them use these. If you have in the school, or can bring there, several pairs of scissors, let them take turns in cutting shapes of different sizes from paper.

Have you any clay in your town that the children can mold into shapes? If you have, let them mold these different solids.

## STEP IV

THE children have been noticing and describing differences in magnitude. Let us now in this Fourth Step lead them to observe the exact differences between different objects.

First, let them discover all the things they know that are alike. Show the class all of the blocks and let one child after another choose the ones that are alike in size and shape.

Make an outline picture of a cube or a sphere on the board and ask some child to make one equal to it. Have the other children observe and tell what he does. This will be hard for the children to do, but their efforts and the criticism of the other children will help them all to realize the difference between equality and inequality.

We can make them appreciate the equality of objects, too, by asking them to build solids like other solids.

We need to use our blocks here. Take those of one shape first, as the prisms, and have several of different lengths ready to use in building. (See illustration on page 38.)

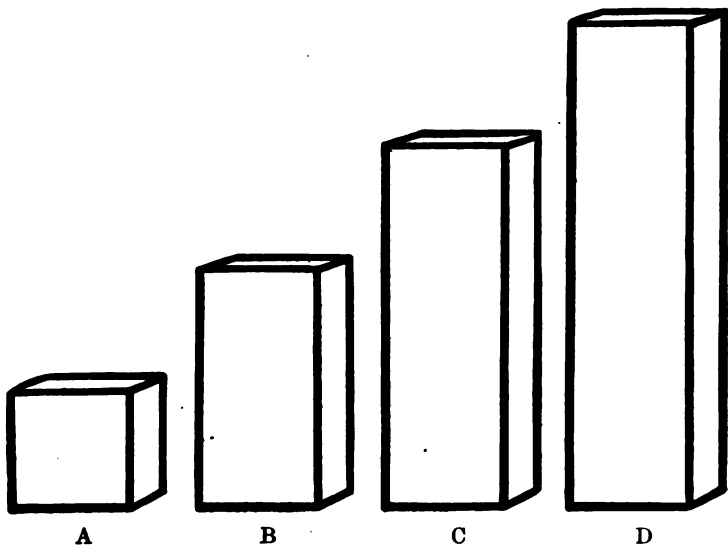
We shall have to name these prisms to distinguish them. Show B to the class and ask if any one can build a prism like it.

How many A's make B?

Can your children answer you correctly,

"Two A's make B"?

We must be particular every day about the English speech of the children. The correct expression in English of what they observe and do is something most important for the children to gain. I do not mean that I wish you to drill the



children over and over on set phrases or on English words. This is parrot work. The English names of the objects that are familiar to them, the children will learn almost without effort on your part or on theirs; and distinct pronunciation of the words, on your part, will help them to pronounce clearly. With each step we have used a few new words or expressions. These the children ought now to use readily, as they need them.

In this step they need as yet only one new word, *equal*. Make them feel the need of the word before you give it to them. Let them build the prism and make it just like B. Then they need to say "2 A's equal B." Here, as everywhere else, if the children gain first the idea, the word to express that idea will mean something to them and, once learned, will stay in their memories.

The pronunciation of the new words, however, is something quite apart from the meaning that they have. A child or a grown person may understand perfectly the idea that the word *equal* expresses, and yet miscall the word. This is because his teacher has not taught him well. My experience is that these children can speak English very well, if we take the trouble to teach them well; and, here as elsewhere, it is the first steps that count the most.

Come back now to the blocks.

Show the block C.

Build other blocks equal to C.

A and B make C.

Are A and B equal?

Can you make C with A's?

How many A's make C?

How many A's make B?

Show the block D.

Build other blocks equal to D.

Let the children choose their own blocks for the work, then let them tell you what blocks they used, and if those blocks were alike.

One child will build A and C.

Another child will build 2 B's.

Another will build 4 A's.

Unless they notice it voluntarily, don't ask them to notice that B is 2 times A, or that C is 3 times A. It is enough, at first, if they see that they all have made solids equal to D.

For another lesson use the different cylinders in the same way.

For another lesson show sphere and hemispheres. Let some child put the two parts together and feel the joy of making the new sphere

Show the parts separately.

Are they alike?

Are they equal?

We call each, half a sphere. How many halves make the sphere?

Show the half relation again with the prisms A and B.

2 A's make B.

A is  $\frac{1}{2}$  of B.

Show them the block D. Have some child build a prism half as large as D.

What is half as large as D?

"B is half as large as D."

In another lesson show them B. Ask for  $\frac{1}{2}$  of B. Ask for  $\frac{2}{2}$  of B. Ask for  $\frac{3}{2}$  of B.

If the  $\frac{2}{2}$  is hard for them, that is enough for once. If they know that easily, go on to  $\frac{3}{2}$ .

Another time build larger than B. Build 2 times as large as B. Can the children do this readily?

Review carefully with the blocks what the children have learned. Let them learn  $\frac{1}{3}$  and  $\frac{2}{3}$ .

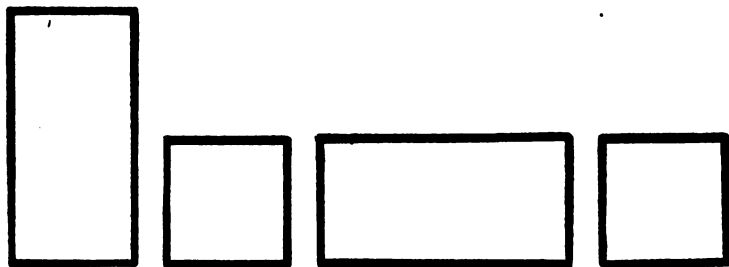
A is what part of C?

B is what part of C?

Show blocks of different size, and have the children compare them and tell the relations of their sizes.

"This prism is two times that one."

"This cube is one half of that prism."



Give groups of 2 or 4 or 6 blocks of the same size and let the children find  $\frac{1}{2}$  of each group.

On other days you may continue this work with lines of different lengths.

Give the children split bamboo sticks 5, 10, and 15 centimeters long, and let them compare their lengths and tell you about them.

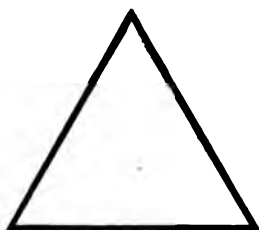
This step has begun definite measuring. The children have measured different shapes and magnitudes by ones that were larger and by ones that were smaller; and they have learned how to express the result of these measurements in English.

This measuring of course has all been done by the eye. We will wait until later before we make definite measurements of time or distance or area by comparing them with some settled unit of measure.

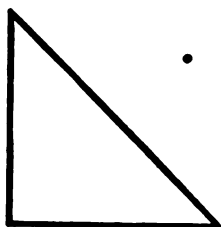
For Seat work here, let the children draw lines and figures illustrating the half and thirds and two times and three times and equal relations. Give them seeds or flowers or leaves or fruits and let them find  $\frac{1}{2}$  or  $\frac{1}{3}$  or 2 times or 3 times the number in the groups given. This work will probably necessitate counting beyond 10. Teach the children then to count to 30 or to 50.







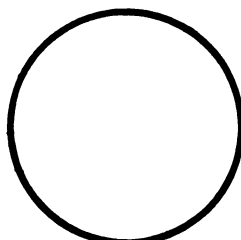
**EQUILATERAL TRIANGLE**



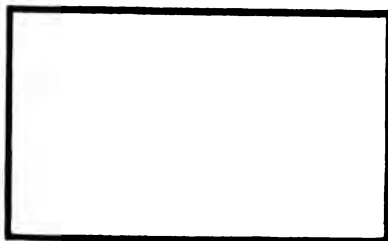
**RIGHT ISOSCELES  
TRIANGLE**



**SQUARE**



**CIRCLE**



**OBLONG**

## STEP V

IN this step we will review the work of the preceding lessons. We will use for this review new objects because we want the children to remember the relations of the numbers, not to connect these relations with any one set of objects.

Show the children the bottom of the cylinder. Will they tell you that it is like a ball or a sphere? It would be natural for them to do so because they have been making circles on the board and on paper to represent spheres. Tell them, "Yes, it is like the picture of a sphere, and we call it a circle."

Have circles made of cardboard. If you can have colored cardboard, so much the more attractive for the children.

Let the children find out by handling the objects the differences between the solid and the plane figure. Ask them to find some circles in the room. Encourage them to make circles on the blackboard and to talk about them.

Who can make a circle?

"José can make a circle."

"Maria can make a circle."

The circles they make will probably be different in size. Let the other children tell you about the different circles.

"Juan's circle is large."

"Carmen's circle is smaller than Maria's circle."

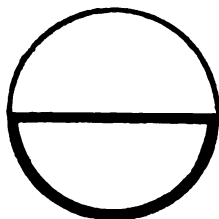
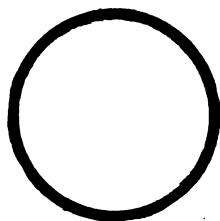
If you have colored chalk, and the children can make circles of different colors, these will give you still more opportunity for conversation.

“José can make a small blue circle.”

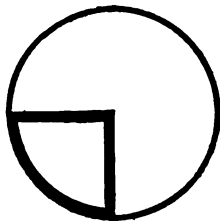
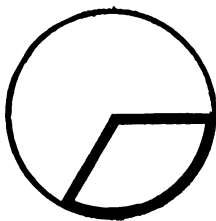
“Maria’s circle is large and red.”

Let each child choose his color and make his own figure, and then have the others describe what he has made.

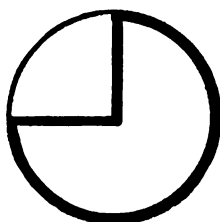
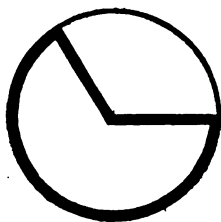
Show them half circles. Can they tell you what they are?



Show them also  $\frac{1}{3}$  of a circle, and  $\frac{1}{4}$  of a circle. Can they tell what part each is of the whole circle?

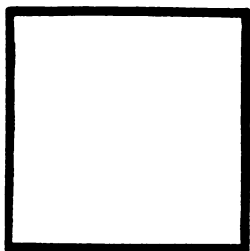


Show them also  $\frac{2}{2}$ ,  $\frac{2}{3}$ ,  $\frac{3}{3}$ ,  $\frac{3}{4}$ , of circles and let them tell you what parts they are of the whole.



Continue this work until the children are quick and accurate in their answers.

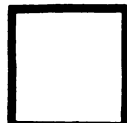
In succeeding lessons take up the same work with squares and oblongs.



A



B



C

Make figures like these on the board.

How many B's in A?

C is what part of B?

How many C's in A?

C is what part of A?

How many C's in B?

B is what part of A?

Work of this kind will help much the measuring idea.

A good class exercise is to have enough rectangles of different heights, or sticks of different lengths, for each child in the class to have one.

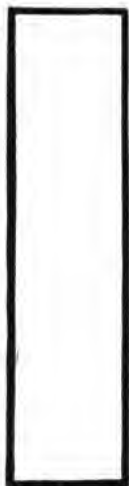
After the rectangles or the sticks have been given out, let each child tell about his figure, by comparing it

1. With a unit that the teacher holds, or makes upon the board.

2. By comparing it with his neighbors' units.

This work gives the children chance for much conversation and encourages their powers of observation.

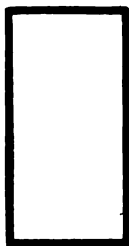
For another day, draw upon the board good-sized rectangles of various heights, like these, and question the children about them:



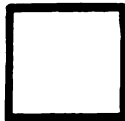
D



C



B



A

If C is 1, what is A?

If C is 1, what is B?

If C is 1, what is D?

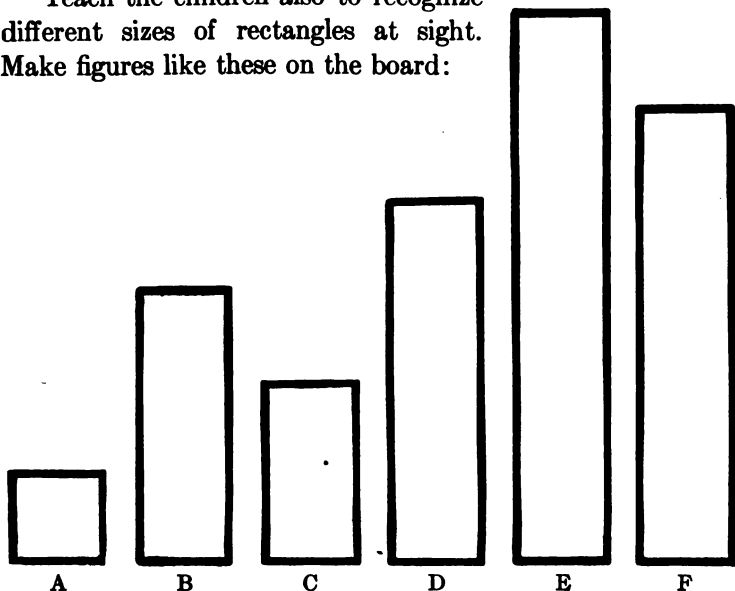
If C is 3, what is A?

If C is 3, what is B?

If C is 3, what is D?

Continue this work with many similar questions.

Teach the children also to recognize different sizes of rectangles at sight. Make figures like these on the board:



Give them A as a unit. A is 1.

What is B?

What is C? etc., etc.

Change the unit. Take C as a unit.

What is A? D? E?

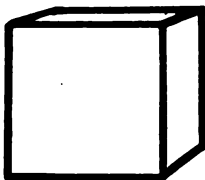
Can they tell you what B is?

Give them drill on this work with lines and sticks as well as with rectangles, until they can recognize quickly the various ratios.

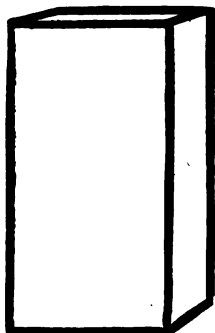
Give review lessons too with solids.



C



A



B

Build a prism like B.

How many A's do you use?

Which is larger, B or A?

How much larger is B than A?

If A is 1, what is B?

If B is 1, what is A?

Build a prism like C.

How many A's do you use?

If A is 1, what is C?

If A is 2, what is C?

If C is 1, what is A?

Build A on B. What does this prism equal?

If A is 1, what is B?

If A is 1, what is C?

If C is 1, what is A? What is B?

Build A on C.

Which is larger, A and C, or B?

If A is 1, what is B?

If A is 1, what is C?

If A is 1, what is A and C?

With blocks and figures of the same size, teach  $\frac{1}{2}$  of 4,  
 $\frac{1}{2}$  of 6,  $\frac{1}{3}$  of 6.



Build up a prism of 6 cubes.

How many cubes are  $\frac{1}{2}$  of the prism?

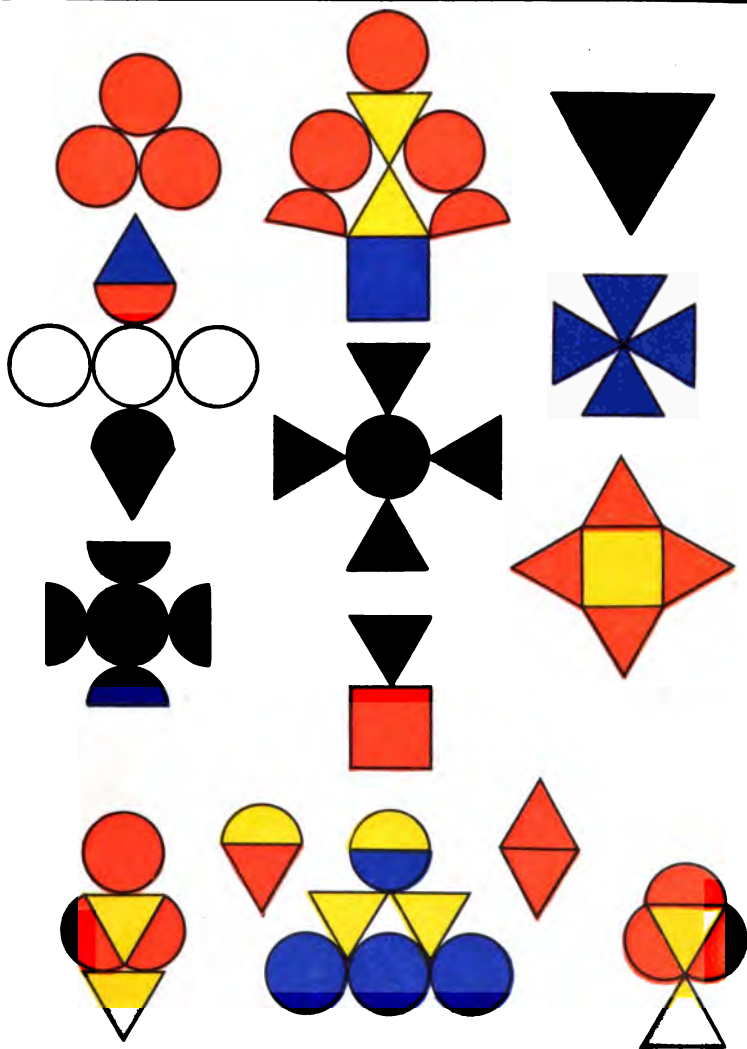
How many cubes are  $\frac{1}{3}$  of the prism?

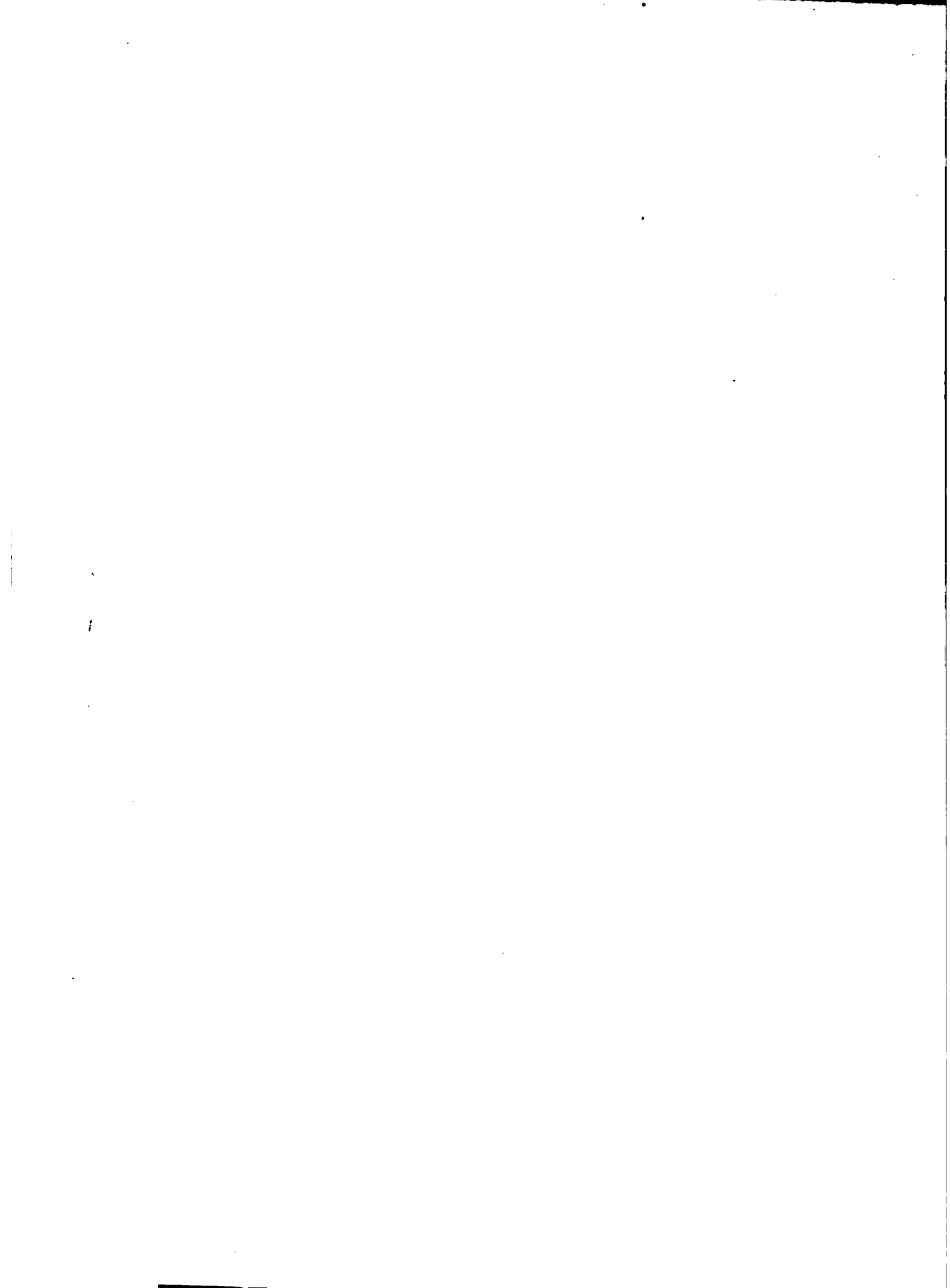


For Seat work have many figures—circles, squares, oblongs, and triangles—and let the children make designs with them.

Encourage the children to make designs of their own.

Review also during this step the Seat work that the children have done previously.





## STEP VI

WE have ascended five steps and we have not even whispered anything about addition or subtraction. I know that you have been thinking for a long while that it was quite time for us to begin this work, and now I am willing to agree with you. I began to plan for teaching addition, though, in the last step, when I asked you to teach your children to recognize at sight the size of different rectangles.

In adding two quantities, there are really three ideas that the child pictures to himself. If he wishes to add 3 and 2, he must realize the quantity 3 and the quantity 2, and then he must be able to picture to himself their sum in the quantity 5. We have shown the children rectangles and prisms and cylinders; and they have learned to recognize at first sight their comparative sizes. They have thus taken a step toward forming a permanent mental picture of the values these figures represent. (See illustration on page 54.)

Show the children C and B and A.

If A is 1, they can tell you at once that B is 3 and C is 4.

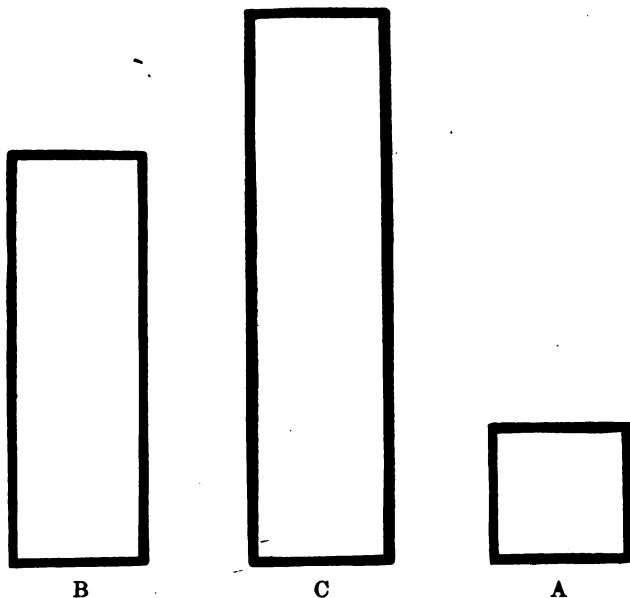
The only remaining step is to combine A and B and prove its identity with 4.

A and B together equal what?

You may teach the children how to write this with the symbols,

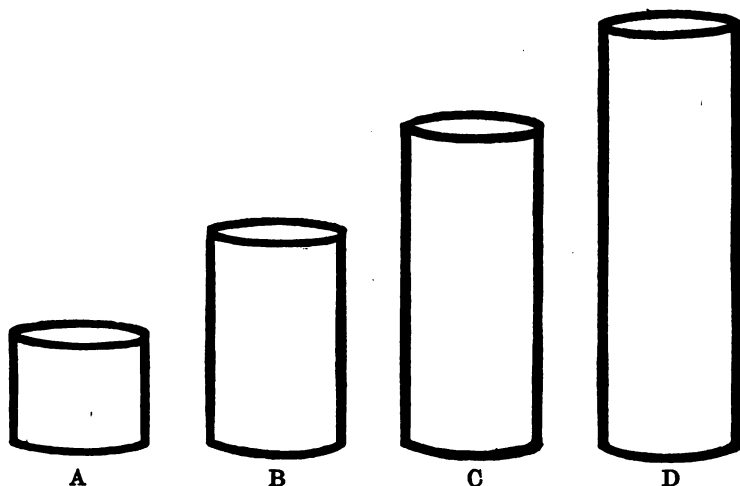
$$3 + 1 = 4$$

When the children understand the thing, they are able to understand the sign we use to represent that thing. They



already know the meaning of equal; the only new process here is the adding of the numbers.

Use the prisms and the cylinders also in teaching addition.



With these four cylinders you can teach many lessons. If you have no cylindrical blocks, you can easily make some from bamboo stalks. Take stalks of the same diameter and cut them evenly in lengths that are 1, 2, 3 and 4 times some common length. Make them, for instance, 5, 10, 15, 20 centimeters high.

Build me a cylinder like D, José. José may use A and C.

Which is larger, A or C?

How much larger is C than A?

How much larger is D than A?

If A is 1, what is D?

If A is 1, what is C?

A and C together make what?

Another child may build with 2 B's.

Have the children state the relative sizes of the cylinders and then tell what sum they make with them.

They have now the three arithmetical expressions:

$$1 + 3 = 4$$

$$2 + 2 = 4$$

$$3 + 1 = 4$$

Let them build 1 on 3, and 3 on 1, and see that they both make 4. Show them this with several combinations, and they will soon realize that order of numbers means nothing in addition.

With these same blocks have the children work out subtraction. After they have built 3 and 1 together and have made the sum 4, let them take away 1 and show the remainder 3. Give them the symbols here, too, and teach them to read the symbols as well as to write them.

$$4 - 1 = 3$$

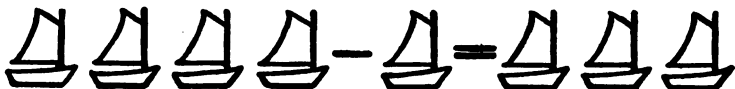
$$4 - 3 = 1$$

$$4 - 2 = 2$$

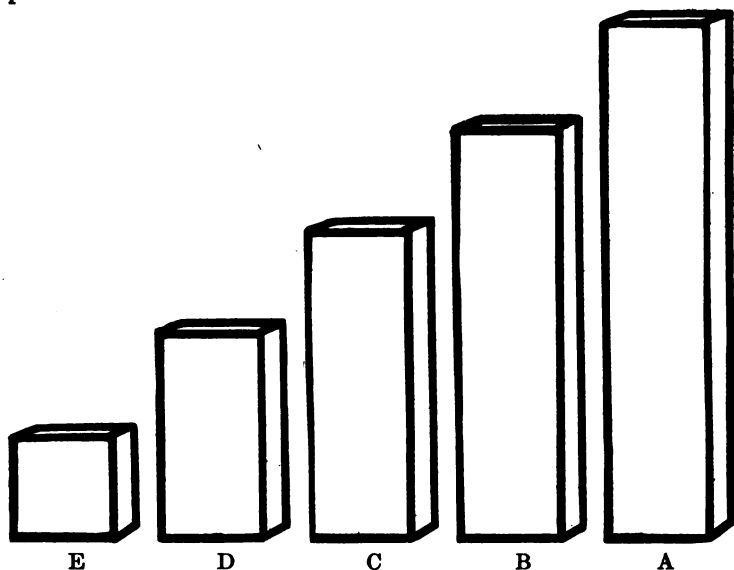
For Seat work let them copy these combinations and illustrate them.

$$2 + 2 = 4$$

$$4 - 1 = 3$$



This copying is not a repetition to teach them over and over the fact  $3+1=4$ . It is practice in writing the figures and signs so that the child's hand work in number may keep pace with his mind work.



For another day's work give them 5 prisms. Have the children in turn build all possible combinations that will equal 5. Then have other children express on the board with symbols what has been done.

They can develop

$$2+3=5$$

$$4+1=5$$

$$2+2+1=5$$

$$1+1+1+1+1=5$$



Work out also the different subtractions from 5.

Teach all the possible combinations with the numbers from 1 to 7.

Give other lessons with blocks and figures and lines of equal size as



Take away 2 circles; how many are left?

How many are 5 circles and 1 circle?

You may teach them these combinations, too, with other objects—flowers and sticks and fruits and seeds.

As soon as the children know well a few of the combinations, give them each day one or two short oral problems:

Juana has 4 bananas. Maria has half as many.

How many bananas has Maria?

José had 3 cents. His father gave him 2 cents.

How many cents had José then?

Do not repeat each problem over and over. Expect the children to listen attentively the first time you say it, and after they have had a second to think in, demand that they give you the answer readily.

The children need training in listening to good English as well as practice in speaking. These problems will drill their listening powers and will also prepare them to express later little problems of their own.

Make the problems always about objects and actions with which the children are familiar. The children understand about giving and losing and picking and selling and buying. They do these every day.

In these first problems give only the number combinations that the children thoroughly understand. When you expect them to hear and answer the conditions of a problem, you are asking them to make a mental picture of the number work of the problem; and this they can do only with combinations that they are thoroughly acquainted with. After the children have learned new number combinations, use those in their problems; but begin the problem work with the combinations that are the simplest and most natural to them.

As the children give you the oral answer to a problem, write the answer on the board in the arithmetic symbols and call their attention to it. This will help to connect the two means of expression.

Juliana has 6 lansones. She gives 3 to her brother.  
How many lansones has Juliana now? 3.

$$6 - 3 = 3$$

Carmen has 4 books. Her teacher gives her another.  
How many books has Carmen now?

$$4 + 1 = 5$$

José has 2 tops, and Juan has three times as many.  
How many tops has Juan?

$$3 \times 2 = 6$$

Tomás has 6 balls, and Domingo has half as many.  
How many has Domingo?

$$\frac{6}{2}=3$$

This is the best way to express  $\frac{1}{2}$  of 6. The regular division with its sign  $\div$ , we will learn by and by.

For drill work give the class three blocks, representing 2 and 3 and 5, for instance. Let the children combine them in all possible ways. For each combination make them express in English what they have done.

2 and 3 are 5.

5 less 3 are 2.

5 less 2 are 3.

Do not tire the children with too long drills like this, but give short daily reviews on the combinations of each number.

On another day give the class a lesson on three rectangles like these:



A



B



C

How much greater is B than A?

What must one add to A to make B?

If A is 1, what is B?

How much greater is 2 than 1?

How much greater is C than A?

What must one add to A to make C?

How much greater is C than B?

What must one add to B to make C?

How much greater is 3 than 1?

How much greater is 3 than 2?

These questions "How much greater?" "How much older?" are always puzzles to the children. The thought is not necessarily a hard thought if we approach it the right way. Do not give the children the subtraction symbols with these questions, but have them get the relation from the blocks and the rectangles, and then express it in words. They may include these in their daily drill work.

5 is 2 greater than 3.

5 is 3 greater than 2.

## STEP VII

STEP VI has taken the children through the combinations of numbers up to 7. There are 37 of these different combinations:

9 in addition,

$1+1$	$1+3$	$1+5$	$2+3$	$3+3$
$1+2$	$1+4$	$2+2$	$2+4$	

20 in subtraction,

$2-1$	$3-3$	$4-4$	$5-4$	$6-3$
$2-2$	$4-1$	$5-1$	$5-5$	$6-4$
$3-1$	$4-2$	$5-2$	$6-1$	$6-5$
$3-2$	$4-3$	$5-3$	$6-2$	$6-6$

5 in multiplication,

$2 \times 1$	$2 \times 3$	$3 \times 2$
$2 \times 2$	$3 \times 1$	

3 in division,

$\frac{4}{2}$	$\frac{6}{2}$	$\frac{6}{3}$
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We cannot expect the children to become well acquainted with these all at once. It will take time to establish friendly and familiar relations with them all. So in Step VII go over

these questions again. Give a little work each day with blocks or objects. You will find that the children will learn some of these combinations very quickly, while they will hesitate over others and will need to be drilled on them.

For new work in this step have the children learn to state problems of their own. These will perhaps be the first connected sentences that they have attempted to say. They will enjoy making the problems, and this work will strengthen both their English and their arithmetic.

Have them form the habit of stating each part in a separate sentence instead of rambling on with "and." These concise statements, I find, make it easier for them to think connectedly.

"Juana went to market. She bought six bananas.  
She gave half of them to her sister. How  
many bananas had Juana left?"

If it helps the children, give them an arithmetical statement on the board, as  $2 + 4 = 6$ , and let them frame problems about it. Encourage as much as possible the story-telling idea.

If you can draw, picture number stories on the blackboard and have them put the story into words.

Picture 5 birds on a fence; show 2 flying away. Some child will frame for you the question and give you the answer.

Picture a coconut tree with fruit on it and other fruit on the ground.

Picture 2 boys spinning tops and 4 boys more flying kites. These pictures will add variety and will heighten their story-telling zeal.

Bring to the class the measures of capacity that you use in your markets and stores.

Have the children use them in the class and find out for themselves how many quarter chupas make the chupa, how many chupas fill the media ganta and the ganta.

Give them problems in buying and selling rice and have them give you similar problems.

For Seat work you can now give them figure problems to complete.

Add

3	4	2	2	4
<u>2</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>2</u>
	1	1		

Subtract or take away

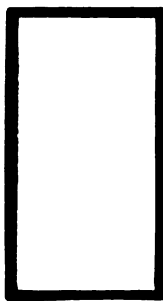
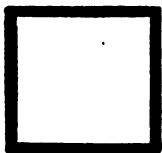
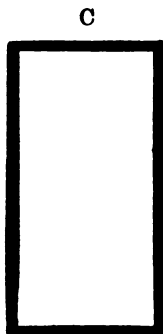
5	6	5
<u>1</u>	<u>2</u>	<u>3</u>

Help them here about their studying. Have blocks or sticks or objects on the desk. If a child is confused about some combination, teach him to come to the desk and work it out for himself with the objects. And in the written work, expect and accept only the best work in neatness and accuracy that the children are capable of accomplishing.

## STEP VIII

FOR Seat work in the preceding step we gave the children examples in addition and subtraction. I should give them here more drill upon these combinations.

Begin the drill by showing them groups of rectangles, like these:





If A is the measure, what is B? C? D? E? F?

How much are A and B together?

How much are C and D together?

How much are E and F?

When you have reviewed the combinations in this way, give the figures alone:

$$\begin{array}{r} 3 \\ 1 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ 2 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ 2 \\ \hline \end{array}$$

and require the sums quickly.

We want the figures  $\frac{2}{3}$  to call up a picture in the child's mind of the two magnitudes and of their sum. And the picture of this work in the child's mind should be so clear that he need never afterward hesitate over these combinations. This work must be done slowly but surely. Five minutes a day spent in drill like this will yield you rich results by the end of the year.

Give combinations of 2 or 3 or 4 figures, only do not allow the sum to reach beyond ten. All possible combinations below 10 are our limits for this year's work. This is a good deal for the little ones to accomplish in one year, if they do it well. We expect no more of our children in the United States in the first year, and here we are working in a new language.

In this step teach the combinations from 6 to 10. There are 62 in all.

17 in addition,

$2+5$	$2+8$	$3+6$	$4+5$	$5+1$	$8+1$
$2+6$	$3+4$	$3+7$	$4+6$	$6+1$	$9+1$
$2+7$	$3+5$	$4+4$	$5+5$	$7+1$	

34 in subtraction,

$7-1$	$7-7$	$8-6$	$9-4$	$10-1$	$10-7$
$7-2$	$8-1$	$8-7$	$9-5$	$10-2$	$10-8$
$7-3$	$8-2$	$8-8$	$9-6$	$10-3$	$10-9$
$7-4$	$8-3$	$9-1$	$9-7$	$10-4$	$10-10$
$7-5$	$8-4$	$9-2$	$9-8$	$10-5$	
$7-6$	$8-5$	$9-3$	$9-9$	$10-6$	

5 in multiplication,

$2 \times 4$	$3 \times 3$	$5 \times 2$
$2 \times 5$	$4 \times 2$	

5 in division,

$\frac{8}{2}$	$\frac{10}{2}$	$\frac{9}{3}$	$\frac{8}{4}$	$\frac{10}{5}$
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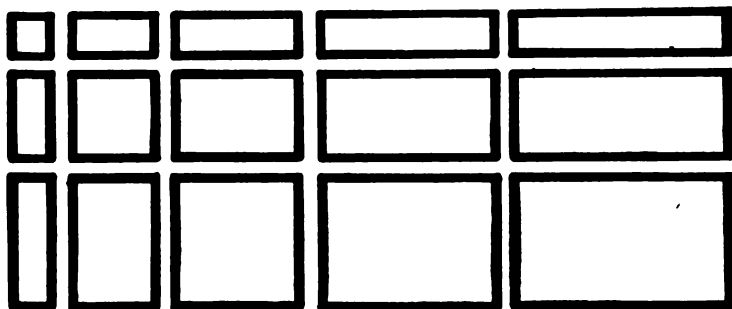
Work out the addition and subtraction combinations as you did the ones below 6.

Build a solid of 7 units. Build another of  $5+2$  units. Compare the two figures and express in words and figures what you find out.

Build solids of 4 and 3 units, and 6 and 1 units, and compare them in the same way. This work gives you all the combinations that produce the sum 7.

In a similar way develop the combinations that produce the other numbers.

For the multiplication and division give figures like these:



Have the children give the size of each one, using the smallest rectangle as a unit. You will use the numbers 12 and 15 here, and you can show their value by adding one unit at a time to the figures you used to represent 10.

Ask questions about these:

How many 2's in 6? in 8? in 10? How many 3's in 9?

3 is what part of 9?

6 is what part of 9?

How many 3's in 6?

How many 3's in 12?

Require correct quick answers from the children.

Continue problems throughout the year. Give problems about these new numbers as well as about the smaller ones. Have the children give you problems of their own.

Write simple problems on the board and have the children read them. In this reading they will learn to recognize the words *two*, *three*, etc.

For Seat work continue the examples begun in Step VII.

## STEP IX

MAKE 12 circles on the board. Have the children count them. If they hesitate after 10, help them to understand the numbers 11 and 12, and teach them to make the figures. We call twelve things a dozen. These are a dozen circles.

How many are half a dozen circles?

How many 6's are there in a dozen?

How many 2's are there in a dozen?

How many 3's?

How many 4's?

How many are half a dozen eggs?

Carmen bought  $\frac{1}{3}$  of a dozen eggs. How many eggs did she buy?

If a dozen chicos cost 10 cents, how much will half a dozen cost?

6 lansones cost 4 cents. How much will a dozen lansones cost?

For Seat work with this lesson give the children

$$\frac{12}{2} = ? \quad \frac{12}{6} = ? \quad \frac{12}{4} = ?$$

$$2 \times ? = 12$$

$$3 \times ? = 12$$

$$4 \times ? = 12$$

and let them work out the problems with their sticks or other objects, and write the answers.

We call this length one inch:



Make a line 2 inches long.

Make a line 3 inches long.

How many are 2 inches and 3 inches?

Draw a line 6 inches long. Let the children do this by sight and then compare their lines with the accurate length to test them.

What is  $\frac{1}{2}$  of 6 inches?

What is  $\frac{1}{3}$  of 6 inches?

What is  $\frac{2}{3}$  of 6 inches?

12 inches together we call a foot.

How many inches in half a foot?

How many inches in one-third of a foot?

Have the children in turn draw an inch on the board. Test the lengths they draw with your measure. Have the children draw a foot. Have them place objects a foot apart on your desk. This work is to test their comprehension of the magnitude *inch* or *foot*. They will need a good deal of this work. They must gain a clear conception of what a foot is before they can go on and realize the length of a yard, or later of a mile.

Have the children stand 3 feet apart, and tell them to call that distance a yard. Have them draw lines a yard long.<sup>1</sup>

Encourage the children to make problems about these measures of length.

Juan walks 4 feet. Carmen walks twice as far. How far does Carmen walk?

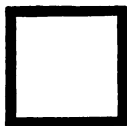
After the children have become familiar with these relations, let them make a table of the measures in order.

12 inches = 1 foot

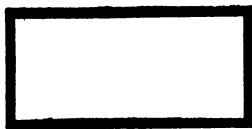
3 feet = 1 yard.

For Seat work, have them draw squares and oblongs that are a certain number of inches each way.

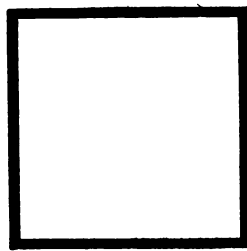
For another lesson make squares and rectangles of definite measure on the board. Ask the children to determine at sight the dimensions of the figures. Then let them compare their relative sizes.



A



B



C

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<sup>1</sup> Have foot rules, made of cardboard if you have no wooden ones, and let the children measure the objects about them, and tell you the length and the width.

"A is 1 inch long and 1 inch wide."

"B is 2 inches long and 1 inch wide."

"B is two times A."

"A is one half of B."

If A is 1, B must be 2, and C must be 4.

Ask them the cost of each, if A costs 1 cent.

If A costs 2 cents, what will B cost? What will C cost?

If A costs 3 cents, what will B cost? What will C cost?

For Seat work, in addition to the examples in addition and subtraction and the work already suggested, write out arithmetic stories with the numbers missing, and have the children copy the stories, insert the numbers, and write the answers.

Maria has ——— cents. She spends ——— cents.

How many cents has she left?

This gives them practice in sentence copying and yet gives them a chance to express number relations of their own choosing.

This work completes the ninth and last step.

## CONCLUSION

STEP IX finishes our year's work in arithmetic. The children can now use the combinations of numbers through 10, and they can understand the application of the various processes in simple problems. They have learned the figures and other written symbols of arithmetic, and so they can put upon paper what they have learned.

In their other lessons they have been learning how to read and write. With this equipment the children will now be able to use a text-book, and so we may send them on to the proud possession of an arithmetic book of their own.

For us teachers, it will be well, I think, to pause a moment, in order to look over the ground we have covered and to sum up the method and purpose of our work. By the measure of what we have done with our class and of why we have done it, we may chart more clearly our future course.

How to use numbers was what we set ourselves to teach our children. What are numbers? Numbers are symbols, we find; and we must look behind the symbols to find what they stand for. Let us question ourselves, What do we mean when we say 5 books? What picture do these two words bring to our minds? We see, and as clearly as if they were before our eyes, a definite number of books, do we not? We are measuring a quantity of books, and we measure them by



telling how many 1 books we have. These words do not measure the quality of the books. They may be good books or very bad ones, expensive or exceedingly cheap. We may describe these qualities in words, but in numbers we can only express facts about their quantity or their size.

So, also, with all other uses of number. Sometimes we count in order some objects, 1, 2, 3, 4, 5 trees or 2, 4, 6 mangoes, or 1 dozen, 2 dozen, 3 dozen eggs. We may use many different units to measure by. We may measure by ones or twos, by halves or dozens, by pounds or gantas, or cavans. But whenever we use a number symbol we are denoting some magnitude.

It is the measure of the *how much* and the *how many* of all the things in the universe, that numbers express. We measure all these things by comparing them with some other thing. Numbers are the symbols, but these relations of magnitude are what we must know and understand. And it is these relations that we wish the children to realize.

The child is a living, active being, and we appeal to his faculties to aid us. We give him objects and lead him to use his powers of observation and find out all the relations that may exist between them. We let him build and take apart, unite and separate the objects. He is gaining experience about the relations of these things. And we let the experience carry the lesson to his mind until he has there a clear mental picture of what he does. And when he has that we encourage him to express it.

The child can express his knowledge in several ways. He can tell us in words that his cube is half the prism, or he can

express the same fact in symbols, or he can draw or cut or mold for you a representation of that relation.

These two things, clearness of mental vision and power of expression, are what we seek above all things to develop in the child. If he sees clearly with his mind, he understands; and if he understands and acts, he has power.

What is the aim of all education? It is to develop power in the individual, is it not? And power is bred of understanding and of action. The man who knows what to do, and does it, is the one who counts wherever he may be—in his home, or in his business, or in his country's service.

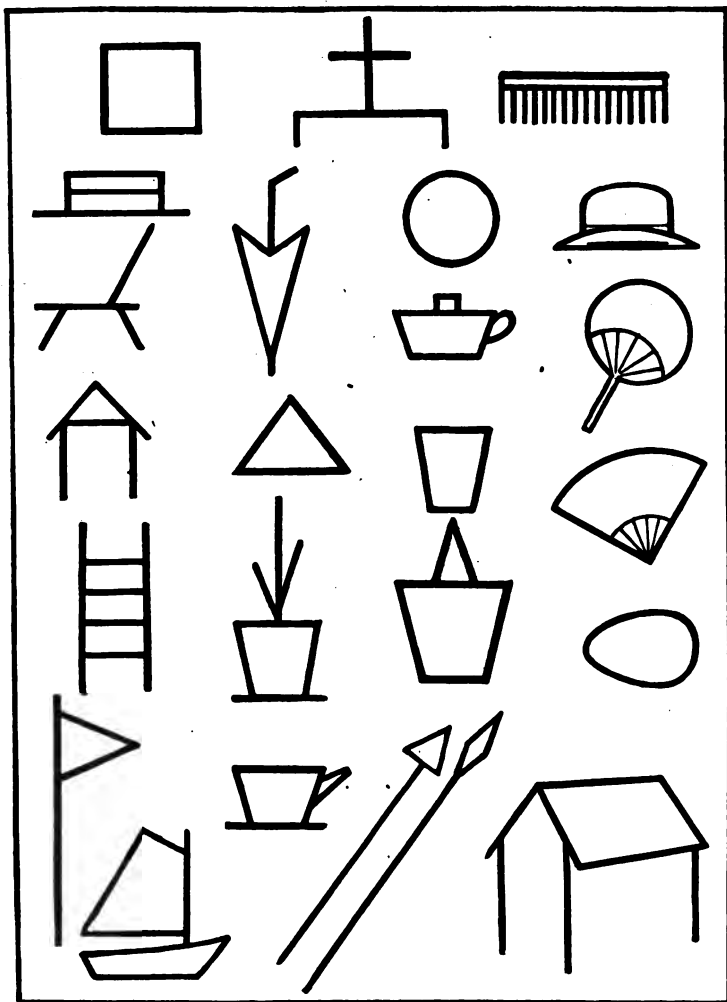
We want our pupils to have something of this power; and our business, here in the beginning classes as much as in the more advanced work, is to help the children to help themselves to gain it.

It is a poor rule, though, that does not work both ways; and if we want to develop power in our pupils, it follows that we must have a little of it ourselves. In our teaching we must know what we want to do, and we must have some plan how to do it. Each day's lesson must have a definite purpose of its own. Its object may be either to lead the children to see some new thing or to establish more firmly in their minds or memories some half-learned truth; but some object and aim it must have, or there will be no progress in our work.

We have had a definite amount of number work that we have wished to bring to the minds of our pupils this year. And while we have been bringing some facts of arithmetic home to the children, we have wished to do other things for them.

The force of habit, of your "costumbre," is one of the strong forces of the world. And people form their own habits, just as truly as they develop their own minds. A man or child forms his own habits; and they become his helpers, or they enslave him and lead him where they will. We wish our pupils to be captains of their habits—to acquire an accuracy in their work, a neatness in its arrangement, a quickness of perception toward new ideas, and a diligence in acquiring them, that will help them all their lives in whatever they may do.

Learning is not all memorizing; teaching is not the hearing of recitations; and the teacher's work is not done until he has stimulated his pupils to action and has implanted in them the desire to use well the information that he has given them.



OUTLINE PICTURES FOR USE IN STEP I.







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